CS 594 – Scientific Computing for Engineers

Homework # 9

Due: April 11, 2012

There are two parts of this homework. The first one is more theoretical and its purpose is for you to gain a more thorough understanding of the ideas discussed in the "Projection and its Importance in Scientific Computing" lecture. The second part is intended to introduce you to some libraries and tools for linear algebra.

PART I:

1. **Prove that the CGS and MGS, as defined on slides #13 and #14, are equivalent in exact arithmetic and that the vectors resulting from the algorithms are orthonormal.**

2. **Show that the algorithm on slide #18 results in Q (step 3) that has orthonormal columns (i.e. $Q^TQ = I$).**
   (Note that this yields an orthogonalization procedure with $A = QL^T$).

3. **Find the projection in $\mathrm{span}\{x, x^3, x^5\}$ of $f(x) = \sin(x)$ on the interval [-1,1] using the inner product and norm as given on slide #36.**

PART II:

This part is to get you started with BLAS, LAPACK, matlab, and the cs594 project.

1. **File chol_qr_it.m implements a QR factorization in matlab. You can try it for example with the following sequence:**

   Start matlab (e.g., on battlecat0.eecs.utk.edu) with
   > matlab -nojvm

   and try the following
   > n=32; m=1000;
   > j=0:n-1;
   > sigma = 2.^(-j);
   > X = randn(n);
   > [u,s,v]=svd(X);
   > norm(X-u*s*v')
   > X=u*diag(sigma)*v';
   > cond(X)
   > [q,r]=chol_qr_it(X);
   > norm(X-q*r)
   > norm(eye(n) – q'*q)
   > tic, [q,r]=chol_qr_it(X); toc
   > tic, [Q,R]=qr(X,0); toc

Report briefly on what is each of these lines doing (you can get help from matlab by typing 'help svd' for example, to find out what is svd).

2. **Implement chol_qr_it.m (in C or Fortran) using calls to LAPACK and BLAS (ATLAS should be installed). Note that you have all the functions that you need (e.g. Q^TQ is in BLAS 3, svd is Lapack's DGESVD, etc. Test on 3 random matrices of size 1000x32, 2000x32, and 3000x32 and report on the norms of X-q*r and I -q'*q as in the matlab code.**