Documentation of ReLIADiff. A C++ Software Package For Real Laplace transform Inversion based on Algorithmic Differentiation *

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Transforms and Inverse Transforms Database

*Accompanying the paper in [1]

Contents

1 Introduction

The files

1. dbLaplace.c,

2. dbInvLaplace.c

contain 89 Laplace Transforms with corresponding Inverses.
Each function is identified by a number so that the user, launching the demo in the directory **DEMO-Functions_From_DATABASE** can choose which function he/she wants to test RELIADIFF on, using the proper number.

Next section lists the Transforms (with the corresponding number) and their Inverses.
The last section shows the meaning of the used symbols.

The database is made of functions coming from [3], [4] and [5].

2 Laplace Transforms/Inverses Database

Each Transform function is of the kind

T$< double >$ $\quad fzXX(T < double > z)$

where XX is the number of the function in the database[1] .

In the next, we will refer to a Laplace Transform as $F(z)$.

Each Inverse function is of the kind

$double \quad gzXX(double)$

where XX is the number of the function in the database.

In the next, we will refer to a Laplace Inverse Transform as $G(t)$.

1. $F(z) = \frac{1}{(z+a)^n (z+b)^n}$ $\qquad a = 3/5,\ b = 5/7,\ n = 5;$

   $G(t) = \frac{\sqrt{\pi}(\frac{t}{b-a})^\nu I_\nu(\frac{1}{2}(b-a)t)}{\Gamma(n) e^{\frac{1}{2}(a+b)t}}$ $\qquad a = 3/5,\ b = 5/7,\ n = 5,\ \nu = n - \frac{1}{2}.$

2. $F(z) = \frac{1}{(z^2+a^2)^n}$ $\qquad a = 3/5, \quad n = 5;$

   $G(t) = \frac{\sqrt{\pi}(\frac{t}{2a})^\nu J_\nu(at)}{\Gamma(n)}$ $\qquad a = 3/5,\ n = 5,\ \nu = n - 1/2.$

3. $F(z) = \frac{1}{(z^2-a^2)^n}$ $\qquad a = 3/5, \quad n = 5;$

   $G(t) = \frac{\sqrt{\pi}(\frac{t}{2a})^\nu I_\nu(at)}{\Gamma(n)}$ $\qquad a = 3/5,\ n = 5,\ \nu = n - 1/2.$

[1] See [2] to know about the class T$< double >$, that extends the C++ type double.

4. $F(z) = \frac{(z-\sqrt{z^2-a^2})^\nu}{\sqrt{z^2-a^2}}$ $\quad a = 3/5, \quad \nu = 3;$

$G(t) = a^n I_n(at)$ $\quad a = 3/5,\ n = 3.$

5. $F(z) = \frac{1}{z^{n+1/2}}$ $\quad n = 5;$

$G(t) = \frac{t^{n-1/2}}{\Gamma(n+1/2)}$ $\quad n = 5.$

6. $F(z) = \frac{1}{(z+a)^n}$ $\quad a = 3/5,\ n = 5;$

$G(t) = \frac{t^{n-1}}{(n-1)!e^{at}}$ $\quad a = 3/5,\ n = 5.$

7. $F(z) = \frac{1}{(z)^n}$ $\quad n = 5;$

$G(t) = \frac{t^{n-1}}{(n-1)!}$ $\quad n = 5.$

8. $F(z) = (\sqrt{z^2+a^2} - z)^n$ $\quad a = 3/5,\ n = 5;$

$G(t) = \frac{na^n J_n(at)}{t}$ $\quad a = 3/5,\ n = 5.$

9. $F(z) = \frac{1}{z^2(z^2+a^2)}$ $\quad a = 3/5;$

$G(t) = \frac{at-\sin(at)}{a^3}$ $\quad a = 3/5.$

10. $F(z) = \frac{1}{(z^2+a^2)^2}$ $\quad a = 3/5;$

$G(t) = \frac{\sin(at)-at\cos(at)}{2a^3}$ $\quad a = 3/5.$

11. $F(z) = \frac{8a^3z^2}{(z^2+a^2)^3}$ $\qquad a = 3/5;$

$G(t) = (1 + a^2t^2)\sin(at) - at\cos(at)$ $\qquad a = 3/5.$

12. $F(z) = \frac{a^2}{z^2(z^2-a^2)}$ $\qquad a = 1/5;$

$G(t) = \frac{1}{a}\sin(at) - t$ $\qquad a = 1/5.$

13. $F(z) = \frac{1}{(z^4-a^4)}$ $\qquad a = 3/5;$

$G(t) = \frac{\sinh(at)-\sin(at)}{2a^3}$ $\qquad a = 3/5.$

14. $F(z) = \frac{(a-b)^n}{(\sqrt{z+a}+\sqrt{z+b})^{2n}}$ $\qquad a = 3/5,\ b = 5/7,\ n = 5;$

$G(t) = \frac{nI_n(1/2(a-b)t)}{te^{1/2(a+b)t}}$ $\qquad a = 3/5,\ b = 5/7,\ n = 5.$

15. $F(z) = \frac{1}{(\sqrt{z+a}+\sqrt{z})^{2\nu}\sqrt{z+a}\sqrt{z}}$ $\qquad a = 3/5,\ \nu = 3;$

$G(t) = \frac{I_n((1/2)at)}{a^n e^{(1/2)at}}$ $\qquad a = 3/5,\ n = 5.$

16. $F(z) = \frac{1}{z(z^2+a^2)}$ $\qquad a = 3/5;$

$G(t) = \frac{1-\cos(at)}{a^2}$ $\qquad a = 3/5.$

17. $F(z) = \frac{z}{(z^2+a^2)(z^2+b^2)}$ $\qquad a = 3/5,\ b = 5/7;$

$G(t) = \frac{\cos(at)-\cos(bt)}{b^2-a^2}$ $\qquad a = 3/5,\ b = 5/7.$

18. $F(z) = \frac{z}{z^4 - a^4}$ $\quad a = 3/5,\ b = 5/7,\ n = 5;$

$G(t) = \frac{\cosh(at) - \cos(at)}{2a^2}$ $\quad a = 3/5.$

19. $F(z) = \frac{1}{(z+a)(z+b)(z+c)}$ $\quad a = 3/5,\ b = 5/7,\ c = -9/7;$

$G(t) = \frac{\frac{b-c}{e^{at}} + \frac{c-a}{e^{bt}} + \frac{a-b}{e^{ct}}}{(a-b)(b-c)(c-a)}$ $\quad a = 3/5,\ b = 5/7,\ c = -9/7.$

20. $F(z) = \frac{1}{z^2};$

$G(t) = t.$.

21. $F(z) = \frac{1}{(z+a)(z+b)}$ $\quad a = 3/5,\ b = 5/7;$

$G(t) = \frac{e^{-at} - e^{-bt}}{b-a}$ $\quad a = 3/5,\ b = 5/7.$

22. $F(z) = \frac{z^2 - 1}{(z^2+1)^2};$

$G(t) = t\cos(t).$

23. $F(z) = \frac{1}{z^2+1};$

$G(t) = \sin(t).$

24. $F(z) = \frac{1}{z^2+z+1};$

$G(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin(\frac{t\sqrt{3}}{2}).$

25. $F(z) = \frac{z^2}{(z^2+a^2)^2}$;

$G(t) = \frac{\sin(at)+at\cos(at)}{2a}$.

26. $F(z) = \frac{1}{z^2-a^2}$;

$G(t) = \frac{\sinh(at)}{a}$.

27. $F(z) = \frac{z}{(z^2+a^2)^2}$;

$G(t) = \frac{t\sin(at)}{2a}$.

28. $F(z) = \frac{z^2-a^2}{(z^2+a^2)^2}$;

$G(t) = t\cos(at)$.

29. $F(z) = \frac{1}{(z+a)^2+b^2}$ $\quad a = 3/5,\ b = 5/7$;

$G(t) = \frac{\sin(bt)}{be^{at}}$ $\quad a = 3/5,\ b = 5/7$.

30. $F(z) = \frac{1}{(z+a)^2}$ $\quad a = 3/5$;

$G(t) = te^{-at}$ $\quad a = 3/5$.

31. $F(z) = \frac{(b^2-a^2)}{\sqrt{z}(z-a^2)(\sqrt{z}+b)}$ $\quad a = 3/5,\ b = 5/7$;

$G(t) = e^{a^2t}(\frac{b\cdot erf(a\sqrt{t})}{a} - 1) + e^{b^2t}erfc(b\sqrt{t})$ $\quad a = 3/5,\ b = 5/7$.

32. $F(z) = \frac{1}{\sqrt{z+a}(z+b)^{3/2}}$ $\qquad a = 3/5,\ b = 5/7;$

$G(t) = e^{-0.5(a+b)t} t \left[I_0(0.5(a-b)t) + I_1(0.5(a-b)t) \right]$ $\qquad a = 3/5,\ b = 5/7.$

33. $F(z) = \frac{1}{z+0.5} + \frac{1}{z^2} + \frac{1}{1+(z+0.2)^2};$

$G(t) = e^{-0.5t} + t + e^{-0.2t} \sin(t).$

34. $F(z) = \frac{1}{\sqrt{z^3}};$

$G(t) = 2\frac{\sqrt{t}}{\sqrt{\pi}}.$

35. $F(z) = \frac{b^2-a^2}{(z-a^2)(\sqrt{z}+b)}$ $\qquad a = 3/5,\ b = 5/7.$

$G(t) = e^{a^2 t}(b - a \cdot erf(a\sqrt{t})) - be^{b^2 t} erfc(b\sqrt{t})$ $\qquad a = 3/4,\ b = 5/7.$

36. $F(z) = \frac{1}{\sqrt{z}(z-a^2)}$ $\qquad a = 3/5;$

$G(t) = \frac{e^{a^2 t} erf(a\sqrt{t})}{a}$ $\qquad a = 3/5.$

37. $F(z) = \frac{1}{(z+b)\sqrt{z+a}}$ $\qquad a = 3/5,\ b = 5/;$

$G(t) = \frac{erf(\sqrt{a-b}\sqrt{t})}{e^{bt}\sqrt{a-b}}$ $\qquad a = 3/5,\ b = 5/7.$

38. $F(z) = \frac{(1-z)^n}{z^{n+3/2}};$

$G(t) = \frac{n! H_{2n+1}(\sqrt{t})}{(2n+1)!\sqrt{\pi}}$ $\qquad n = 5,\ H : Hermite\ polynomial.$

39. $F(z) = \frac{1}{z};$

$G(t) = 1.$

40. $F(z) = \frac{1}{1+2z};$

$G(t) = 0.5e^{-0.5t}.$

41. $F(z) = \frac{z}{(z-1)^2};$

$G(t) = (1+t)e^t.$

42. $F(z) = \frac{1}{z-2};$

$G(t) = e^{2t}.$

43. $F(z) = \frac{z}{(z+a)(z+b)} \quad a = 3/5,\ b = 5/7;$

$G(t) = \frac{ae^{-at} - be^{-bt}}{a-b} \quad a = 3/5,\ b = 5/7.$

44. $F(z) = \frac{z}{z^2+a^2} \quad a = 3/5;$

$G(t) = \cos(at) \quad a = 3/5.$

45. $F(z) = \frac{z+a}{(z+a)^2+b^2} \quad a = 3/5,\ b = 5/7;$

$G(t) = \cos(bt)e^{-at} \quad a = 3/5,\ b = 5/7.$

46. $F(z) = \frac{z}{z^2-a^2}$ $a = 3/5;$

$G(t) = \cosh(at)$ $a = 3/5.$

47. $F(z) = \frac{1}{\sqrt{z^2+a^2}}$ $a = 3/5;$

$G(t) = J_0(at)$ $a = 3/5.$

48. $F(z) = \arctan(\frac{k}{z})$ $k = 9/11;$

$G(t) = \frac{\sin(kt)}{t}$ $k = 9/11.$

49. $F(z) = \frac{1}{\sqrt{z}(\sqrt{z}+a)};$

$G(t) = e^{a^2t}erfc(a\sqrt{t}).$

50. $F(z) = \frac{\sqrt{z+2a}-\sqrt{z}}{\sqrt{z+2a}+\sqrt{z}}$ $a = 3/5;$

$G(t) = \frac{I_1(at)}{te^{at}}$ $a = 3/5.$

51. $F(z) = (\frac{z-1}{z})^n$ $n = 5;$

$G(t) = L_n(t)$ $n = 5,\ L_n : Laguerre\,polynomial.$

52. $F(z) = \frac{1}{z^k}$ $k = 9/11;$

$G(t) = \frac{t^{k-1}}{\Gamma(k)}$ $k = 9/11.$

53. $F(z) = \frac{1}{(z+a)^k}$ $a = 3/5,\ k = 9/11;$

$G(t) = \frac{t^{k-1}}{\Gamma(k)e^{at}}$ $a = 3/5,\ k = 9/11.$

54. $F(z) = \frac{1}{\sqrt{z}};$

$G(t) = \frac{1}{\sqrt{\pi t}}$ $a = 3/5,\ k = 9/11.$

55. $F(z) = \frac{z}{\sqrt{(z+a)^3}}$ $a = 3/5;$

$G(t) = \frac{1-2at}{\sqrt{\pi t}e^{at}}$ $a = 3/5.$

56. $F(z) = \sqrt{z+a} - \sqrt{z+b}$ $a = 3/5,\ b = 5/7.$

$G(t) = \frac{e^{-bt}-e^{-at}}{2\sqrt{\pi}\sqrt{t^3}}$ $a = 3/5,\ b = 5/7.$

57. $F(z) = \frac{1}{\sqrt{z}+a}$ $a = 3/5;$

$G(t) = \frac{1}{\sqrt{\pi t}} - ae^{a^2 t}erfc(a\sqrt{t})$ $a = 3/5.$

58. $F(z) = \frac{(1-z)^n}{z^{n+1/2}};$

$G(t) = \frac{n!H_{2n}(\sqrt{t})}{(2n)!\sqrt{\pi t}}$ $a = 3/5,\ H_{2n} : Hermite\, polynomial.$

59. $F(z) = \frac{1}{\sqrt{(z+1)}};$

$G(t) = \frac{1}{\sqrt{\pi t}}e^{-t}.$

60. $F(z) = \frac{1}{z}\left(1 - \frac{\ln(z+1)}{z}\right);$

$G(t) = 1 - E_1(t).$

61. $F(z) = \frac{\ln z}{z};$

$G(t) = -\gamma - \ln t.$

62. $F(z) = \frac{\ln(1+z)}{z};$

$G(t) = E_1(t).$

63. $F(z) = \ln\left(\frac{z+a}{z+b}\right)$ $\quad a = 3/5,\ b = 5/7;$

$G(t) = \frac{e^{-bt} - e^{-at}}{t}$ $\quad a = 3/5,\ b = 5/7.$

64. $F(z) = \ln\left(\frac{z^2+a^2}{z^2}\right)$ $\quad a = 3/5;$

$G(t) = \frac{2(1-\cos(at))}{t}$ $\quad a = 3/5.$

65. $F(z) = \ln\left(\frac{z^2-a^2}{z^2}\right)$ $\quad a = 3/5;$

$G(t) = \frac{2(1-\cosh(at))}{t}$ $\quad a = 3/5.$

66. $F(z) = \frac{\sqrt{z+2a}}{\sqrt{z}} - 1$ $\quad a = 3/5;$

$G(t) = \frac{a[I_0(at)+I_1(at)]}{e^{at}}$ $\quad a = 3/5.$

67. $F(z) = \frac{e^{k/z}}{z^{\mu}}$ $\mu = 4,\ k = 9/11;$

$G(t) = (\frac{t}{k})^{(\mu-1)/2} I_{\mu-1}(2\sqrt{kt})$ $\mu = 4,\ k = 9/11.$

68. $F(z) = \frac{e^{k/z}}{z^{3/2}}$ $k = 9/11;$

$G(t) = \frac{\sinh(2\sqrt{kt})}{\sqrt{\pi k}}$ $k = 9/11.$

69. $F(z) = \frac{e^{k/z}}{\sqrt{z}}$ $k = 9/11;$

$G(t) = \frac{\cosh(2\sqrt{kt})}{\sqrt{\pi t}}$ $k = 9/11.$

70. $F(z) = e^{-k\sqrt{z}}$ $k = 9/11;$

$G(t) = \frac{ke^{k^2/4t}}{2\sqrt{\pi t^3}}$ $k = 9/11.$

71. $F(z) = \frac{1}{z^{n+1/2}e^{k\sqrt{z}}}$ $k = 9/11;$

$G(t) = \int_0^t \frac{1}{\sqrt{\pi\tau}} e^{-k^2/4\tau} \frac{(t-\tau)^{n-1}}{(n-1)!} d\tau$ $\mu = 4,\ k = 9/11.$

72. $F(z) = \frac{1}{z^{3/2}e^{k\sqrt{z}}}$ $k = 9/11;$

$G(t) = \frac{2\sqrt{t}e^{-k^2/4t}}{\sqrt{\pi}} - k \cdot erfc(\frac{k}{2\sqrt{t}})$ $\mu = 4,\ k = 9/11.$

73. $F(z) = \frac{a}{z(\sqrt{z}+a)e^{k\sqrt{z}}}$ $\qquad k = 9/11;$

$G(t) = -e^{ak}e^{a^2t}erfc\left(a\sqrt{t} + \frac{k}{\sqrt{t}}\right) + erfc\left(\frac{k}{2\sqrt{t}}\right)$ $\qquad k = 9/11.$

74. $F(z) = \frac{1}{ze^{k\sqrt{z}}}$ $\qquad k = 9/11;$

$G(t) = erfc(\frac{k}{2\sqrt{t}})$ $\qquad \mu = 4,\ k = 9/11.$

75. $F(z) = \frac{1}{z+0.5}e^{-\frac{2z-1}{2z+1}};$

$G(t) = e^{-(1+t/2)}I_0(2\sqrt{t}).$

76. $F(z) = \frac{1}{\sqrt{z}(\sqrt{z}+a)e^{k\sqrt{z}}}$ $\qquad k = 9/11;$

$G(t) = e^{ak}e^{a^2t}erfc\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right)$ $\qquad k = 9/11.$

77. $F(z) = \frac{1}{\sqrt{z^2+a^2}e^{k(\sqrt{z^2+a^2}-z)}}$ $\qquad a = 3/5,\ k = 9/11;$

$G(t) = J_0(a\sqrt{t^2+2kt})$ $\qquad a = 3/5,\ k = 9/11.$

78. $F(z) = \frac{1}{\sqrt{z}e^{k\sqrt{z}}}$ $\qquad k = 9/11;$

$G(t) = -J_0(a\sqrt{t^2+2kt})$ $\qquad k = 9/11.$

79. $F(z) = \frac{1}{(\sqrt{z}+a)e^{k\sqrt{z}}}$ $\qquad a = 3/5,\ k = 9/11;$

$G(t) = \frac{e^{-k^2/4t}}{\sqrt{\pi t}} - ae^{ak}e^{a^2t}erfc\left(a\sqrt{t} + \frac{k}{\sqrt{t}}\right)$ $\qquad a = 3/5,\ k = 9/11.$

80. $F(z) = \frac{1}{\sqrt{z+1}} e^{-\frac{z}{1+z}}$;

$G(t) = \frac{1}{\sqrt{\pi}\sqrt{t}} e^{-(1+t)} \cosh(2\sqrt{t})$.

81. $F(z) = \frac{1}{z^{\mu} e^{k/z}}$ $\qquad k = 9/11$;

$G(t) = (\frac{t}{k})^{(\mu-1)/2} J_{\mu-1}(2\sqrt{kt})$ $\qquad k = 9/11$.

82. $F(z) = \frac{1}{z^{3/2} e^{k/z}}$ $\qquad k = 9/11$;

$G(t) = \frac{\sin(2\sqrt{kt})}{\sqrt{\pi}\sqrt{t}}$ $\qquad k = 9/11$.

83. $F(z) = \frac{1}{z e^{k/z}}$ $\qquad k = 9/11$;

$G(t) = J_0(2\sqrt{kt})$ $\qquad k = 9/11$.

84. $F(z) = \frac{1}{\sqrt{z} e^{k/z}}$ $\qquad k = 9/11$;

$G(t) = \frac{\cos(2\sqrt{kt})}{\sqrt{\pi}\sqrt{t}}$ $\qquad k = 9/11$.

85. $F(z) = \frac{1}{z(z+1)} \left[\frac{1}{2z} - \frac{e^{-2z}}{1-e^{-2z}} \right]$;

$G(t) = \frac{1}{2} + e^{-t} \left[\frac{1}{2} - \frac{e^2}{e^2-1} \right] - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi t) - \arctan(n\pi))}{n\sqrt{n^2\pi^2+1}}$.

86. $F(z) = \frac{(100z-1)\sinh(\sqrt{z}/2)}{z[z\sinh(\sqrt{z}) + \sqrt{z}\cosh(\sqrt{z})]}$;

$G(t) = -\frac{1}{2} + \sum_{n=1}^{\infty} \left(100 + \frac{1}{b_n^2}\right) \frac{2b_n \sin(b_n/2) e^{-b_n^2 t}}{(2+b_n^2)\cos(b_n)}$ $\qquad b_n \tan(b_n) = 1$.

87. $F(z) = \frac{1}{z} e^{-r\sqrt{\frac{z(1+z)}{1+cz}}} \qquad r = 0.5, \quad c = 0.4;$

$G(t) = \frac{1}{2} +$

$+\frac{1}{\pi}\int_0^\infty \left[e^{[-rm\sqrt{u/2}(\cos(\theta)-\sin(\theta)]} \; \sin[tu - rm\sqrt{u/2}(\cos(\theta)-\sin(\theta))\right] \frac{du}{u}$

$m = \left[\frac{1+u^2}{1+c^2u^2}\right]^{1/4}, \quad 2\theta = \arctan(u) - \arctan(cu), \quad c = 0.4, \quad r = 0.5.$

88. $F(z) = \frac{e^{-2\Psi}}{z}, \quad \cos(\Psi) = \sqrt{1 + z^2 + z^4/16};$

$G(t) = 1 - \frac{1}{\pi}\int_0^{u_1} [(\sin(ut+2k) - \sin(ut-2k)] \frac{du}{u} +$

$+\frac{1}{\pi}\int_{u_2}^{4} [(\sin(ut+2k) - \sin(ut-2k)] \frac{du}{u}$

$\cos(k) = \frac{1}{4}\sqrt{(u_1^2 - u^2)(u_2^2 - u^2)}, \quad u_1 = 2\sqrt{2-\sqrt{3}}, \quad u_2 = \sqrt{2+\sqrt{3}}.$

89. $F(z) = \frac{z-\sqrt{z^2-c^2}}{\sqrt{z}\sqrt{z^2-c^2}\sqrt{z-N\sqrt{z^2-c^2}}};$

$G(t) = \frac{2}{\pi}\int_0^c \cosh(tu)\frac{u\sqrt{(R+u)/2}+\sqrt{c^2-u^2}\sqrt{(R-u)/2}}{R\sqrt{c^2-u^2}\sqrt{u}} +$

$+\frac{2}{\pi}\int_0^b \frac{u-\sqrt{c^2+u^2}}{\sqrt{u}\sqrt{c^2+u^2}\sqrt{N\sqrt{c^2+u^2}-u}} \cos(tu)du$

$R = \sqrt{u^2 + N^2(c^2-u^2)}, \quad b = \sqrt{(1-N)/(1+N)},$

$c = (1-N)/N, \quad N = 0.5.$

3 Remarks on Symbols

Some remarks about symbols used in functions definition.

Gamma function $\Gamma(t)$, subject to t not being a negative integer or zero:

$$\Gamma(t) = \int_0^\infty e^{-x} x^{t-1} dx$$

Bessel Function of the First Kind:

- regular cylindrical Bessel function of zeroth order, $J_0(z)$.
- regular cylindrical Bessel function of order n, $J_n(t)$.
- regular cylindrical Bessel function of fractional order ν, $J_\nu(t)$.

$$J_\nu(z) = \left(\frac{1}{2}\right)^\nu \sum_{k=0}^{\infty} (-1)^k \frac{(\frac{1}{4}z^2)^k}{k!\Gamma(\nu+k+1)} \quad arg(z) < \pi, \quad \nu \in \Re$$

Modified Bessel Function of the First Kind:

- regular modified cylindrical Bessel function of zeroth order, $I_0(t)$.
- regular modified cylindrical Bessel function of first order, $I_1(t)$.
- regular modified cylindrical Bessel function of order n, $I_n(t)$.
- regular modified Bessel function of fractional order ν, $I_\nu(t)$, for $t > 0$, $\nu > 0$.

$$I_\nu(t) = i^{-\nu} J_\nu(it), \quad t \in \Re, \quad t \geq 0, \quad \nu \in \Re$$

Error Function $erf(t)$:

$$erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dx$$

Complementary Error Function $erfc(t)$:

$$erfc(t) = 1 - erf(t) = \frac{2}{\sqrt{\pi}} \int_t^\infty e^{-x^2} dx$$

Dawson's integral for t:

$$I(t) = e^{-t^2} \int_0^t e^{y^2} dy = \frac{1}{2}\sqrt{\pi} e^{-t^2} erf(t)$$

used to compute:

$$g(t) = \frac{erf(\sqrt{a-b}\sqrt{t})}{\sqrt{a-b}e^{bt}} = e^{-bt} \frac{I(t)\frac{2}{\sqrt{\pi}e^{(\sqrt{b-a}\sqrt{t})^2}}}{\sqrt{a-b}}$$

since

$$erf(\sqrt{a-b}\sqrt{t}) = erf(i\sqrt{b-a}\sqrt{t}) = I(t)\frac{2}{\sqrt{\pi}}e^{(\sqrt{b-a}\sqrt{t})^2}$$

The Exponential Integral $E_1(t)$

$$E_1(t) = \int_1^\infty \frac{e^{-tx}}{t}dx$$

Some used constants:

- Euler-Mascheroni constant γ;
- Greek Pi constant π.

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